Indian Statistical Institute, Bangalore
B. Math (II), First semester 2017-2018
Backpaper Examination : Statistics (I)

Date: 04-01-2018 **Maximum Score 50** **Duration: 3 Hours**

1. Consider the following data set

023	037	070	065	162	050
064	086	057	041	068	020
108	089	025	126	044	197
058	018	013	052	090	044
161	022	082	035	050	174

   (a) Make stem and leaf plots of these data.

   (b) Find the sample mean $\overline{X}$.

   (c) Find $100p$-th percentile for $p = 0{\cdot}25$.

   (d) Find the median M and the first quartile Q_1.

   (e) Draw the box plot and identify the outliers.

   (f) Decide on the trimming fraction just enough to eliminate the outliers and obtain the trimmed median M_{T}.

   (g) Explain (need not compute) how to obtain the trimmed mean $\overline{X}_{\text{T}}$ and the trimmed standard deviation S_{T}.

   (h) Between the box plot and the stem and leaf plot what do they tell us about the above data set? In very general terms what can you say about the population from which the data have arrived? $[3+2+1+2+4+2+3+3=20]$

2. Let $X_1,\ X_2, \cdots, X_n$ be a random sample from the distribution whose *pmf* is proportional to

$$\begin{aligned} g(x|\theta) &= \theta^x,\ x = 1, 2, \cdots; \\ &= 0 \text{ otherwise }; 0 < \theta < 1. \end{aligned}$$

   Find *method of moments (MOM) estimator* for θ. Also find *maximum likelihood estimator (MLE)* for θ. $[5+5=10]$

3. If $X_1,\ X_2$ are independent $Gamma(a, \lambda)$ and $Gamma(b, \lambda)$ random variables, λ, a and b all positive, then find the distribution of $Y = \frac{X_1}{X_2}$. Based on $Y_1, Y_2, \cdots, Y_n$; a random sample from the distribution of Y, find *MLE or* MOM *estimator* for λ. If $\lambda > 0$ and a, b positive integers; were all known then explain how you would generate observations on Y using a *Direct Method*.

   $[8+2+6=16]$

4. Let $X_1,\ X_2, \cdots, X_n$ be *a random sample* from $N(0, \sigma^2)$. Let $Y = \frac{X_1}{\sqrt{\sum_{i=2}^{m+1} X_i^2}}$, $2 \leq m < n$. Find the distribution of Y.

   $[10]$